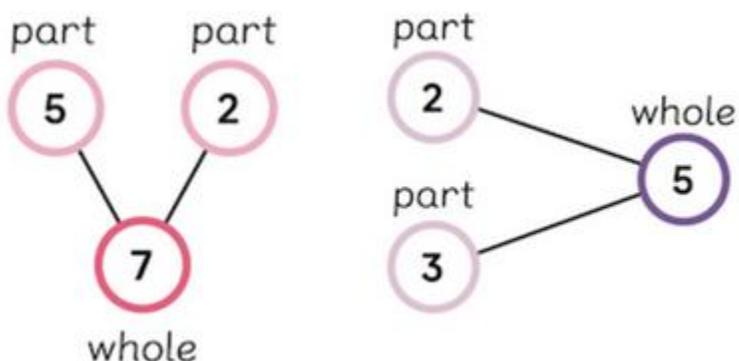**Mathematics written methods at the Spinney**

Written methods for addition of whole numbers

Stage 1: Part/Whole Diagrams.

These are a valuable first step when the children are beginning to learn how to record their work.

Stage 2: The empty number line

The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and units separately, starting with the units. Children need to be able to partition numbers in ways other than into tens and units to help them make multiples of ten by adding in steps. The empty number line helps to record the steps on the way to calculating the total. Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

$8 + 7 = 15$

$48 + 36 = 84$

or:

Stage 3: Partitioning

The next stage is to record mental methods using partitioning. Add the units and then the tens to form partial sums and then add these partial sums. Partitioning both numbers into tens and units mirrors the column method where units are placed under units and tens under tens. This also links to mental methods.

Record steps in addition using partitioning:

47 + 76 = 47 + 70 + 6 = 117 + 6 = 123
47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123

Partitioned numbers are then written under one another:

$$
\begin{array}{lll}
47 & = & 7 + 6 & = & 13 \\
\underline{76} & & 40 + 70 & = & +\underline{110} \\
& & & & 123
\end{array}
$$

Stage 4: Expanded method in columns

Move on to a layout showing the addition of the tens to the tens and the units to the units separately, ask them to start by adding the unit digits first always.

The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'.

The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Write the numbers in columns.

Adding the units first:

$$
\begin{array}{r}
47 \\
+ \ 76 \\
\hline
13 \\
\underline{110} \\
123
\end{array}
$$

Stage 5: Column method

<u>This is the method that the children need to learn and use.</u>

In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.

Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.

```
  47        258        366
+ 76      +  87       +458
─────     ─────       ─────
 123       345         824
 11         11          11
```

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

# Written methods for subtraction of whole numbers

Stage 1: Using the empty number line

The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 - 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten. The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47. With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 - 12, 86 - 77 or 43 - 28.

The notes below give more detail on the counting-up method using an empty number line.

The counting-up method

The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ? = 74 mentally.
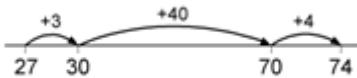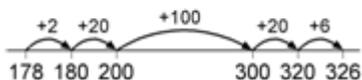

Or:

With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ? = 200 and 200 + ? = 326 mentally.
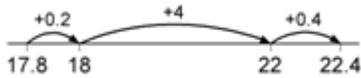
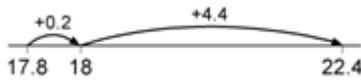The most compact form of recording remains reasonably efficient.

Or:

The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed.
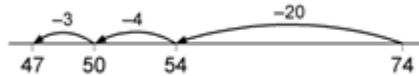
+0.2   +4   +0.4

17.8   18   22   22.4

Or:

+0.2   +4.4

17.8   18   22.4

The counting down method

Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

15 - 7 = 8

−2   −5

8   10   15

74 - 27 = 47 worked by counting back:

−3   −4   −20

47   50   54   74

The steps may be recorded in a different order:

−20   −3   −4

47   67   70   74

or combined:

−23   −4

47   70   74

Stage 2: Partitioning

Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 - 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.

Subtraction can be recorded using partitioning:

74 - 27 = 74 - 20 - 7 = 54 - 7 = 47
74 - 27 = 70 + 4 - 20 - 7 = 60 + 14 - 20 - 7 = 40 + 7

This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.
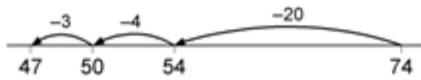
Stage 3: Expanded layout, leading to column method

This is the method that the children need to learn and use.

Partitioning the numbers into tens and units and writing one under the other mirrors the column method, where ones are placed under units and tens under tens. This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills. The expanded method leads children to the more compact method so that they understand its structure and efficiency.

Partitioned numbers are then written under one another:

Example: 74 - 27

The expanded method for three-digit numbers

Example: 563 – 241

Start by subtracting the units, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Example: 563 – 271, adjustment from the hundreds to the tens, or partitioning the hundreds

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten.

Example: 563 − 278, adjustment from the hundreds to the tens and the tens to the units

```
  ⁴ ¹⁵ ¹³
  5̷ 6̷ 3̷
- 2 7 8
─────────
  2 8 5
```

Here both the tens and the unit digits to be subtracted are bigger than both the tens and the unit digits you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

Example: 503 − 278, dealing with zeros when adjusting

```
  ⁴ ⁹ ¹³
  5̷ 0̷ 3̷
- 2 7 8
─────────
  2 2 5
```

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

Written methods for multiplication of whole numbers

Stage 1: Mental multiplication using partitioning

Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.

Informal recording in Year 4 might be:

```
        43
 40  +   3
  │       │   × 6
  ↓       ↓
240  +  18  = 258
```

Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7:

```
ooooooo    ooooo...oo
ooooooo    ooooo...oo
ooooooo    ooooo...oo
```

$7 \times 3 = (5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$

Stage 2: The grid method

As a staging post, an expanded method which uses a grid can be employed. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.

It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

$38 \times 7 = (30 \times 7) + (8 \times 7) = 210 + 56 = 266$

×	7
30	210
8	56
	266

×	30 + 8
	7
	210
	56
	266

The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 210 and 56.

Stage 3: Expanded short multiplication

The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.

Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.

```
  38
X 7
  56
210
266
```

Stage 4: Short multiplication

The recording is reduced further, with carry digits recorded below the line.

```
  38
×  7
266
  5
```

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.

Stage 5: Two-digit by two-digit products

- Extend to TU × TU, asking children to estimate first.
- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.

As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.

56 × 27 is approximately 60 × 30 = 1800.

×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

		50	6	
×		20	7	
		1000	350	1350
		120	42	162
				1512
				1

The aim is for all children to use the following long multiplication method for TU × TU.

56 × 27 is approximately 60 × 30 = 1800.

```
    56
x  27
   392        7 x 56
  1120        20 x 56
  1512
```

Stage 6: Three-digit by two-digit products

Extend to HTU × TU asking children to estimate first.

Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.

Again, the carry digits in the partial products are usually carried mentally.

286 × 29 is approximately 300 × 30 = 9000.

```
   286
x  29
  2574        9 x 286
  5720        20 x 286
  8294
  1
```

Written methods for division of whole numbers

Stage 1: Mental division using partitioning

Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient.

Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention.

Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.

Another way to record is in a grid, with links to the grid method of multiplication.

As the mental method is recorded, ask: 'How many sevens in seventy?' and: 'How many sevens in fourteen?'

Stage 2: Short division of TU ÷ U

- 'Short' division of TU ÷ U can be introduced as a more compact recording of the mental method of partitioning.
- Short division of two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.
- For most children this will be at the end of Year 4 or the beginning of Year 5.

The accompanying patter is 'How many threes divide into 80 so that the answer is a multiple of 10?' This gives 20 threes or 60, with 20 remaining. We now ask: 'What is 21 divided by three?' which gives the answer 7.

For 81 ÷ 3, the dividend of 81 is split into 60, the highest multiple of 3 that is also a multiple 10 and less than 81, to give 60 + 21.Each number is then divided by 3.

$$81 \div 3 = (60 + 21) \div 3$$
$$= (60 \div 3) + (21 \div 3)$$
$$= 20 + 7$$
$$= 27$$

The short division method is recorded like this:

$$\begin{array}{r} 27 \\ 3\overline{)8\,{}^21} \end{array}$$

The carry digit '2' represents the 2 tens that have been exchanged for 20 units. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. In second it is written as a superscript.

The 27 written above the line represents the answer: 20 + 7,or 2 tens and 7 units.

Stage 3: 'Expanded' method for HTU ÷ U

- This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.
- For TU ÷ U there is a link to the mental method.
- As you record the division, ask: 'How many nines in 90?' or 'What is 90 divided by 9?'
- Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same.
- This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.
- Chunking is useful for reminding children of the link between division and repeated subtraction.

However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.

$$
\begin{array}{r}
6\overline{)196} \\
-\underline{60} \quad 6 \times 10 \\
136 \\
-\underline{60} \quad 6 \times 10 \\
76 \\
-\underline{60} \quad 6 \times 10 \\
16 \\
-\underline{12} \quad 6 \times \underline{2} \\
4 \qquad 32 \\
\text{Answer:} \quad 32\,\text{R}\,4
\end{array}
$$

Stage 4: Short division of HTU ÷ U

- 'Short' division of HTU ÷ U is the key method that the children need to learn. No chunking is involved since the links are to partitioning, not repeated subtraction.
- The accompanying pattern is 'How many threes in 290?' (the answer must be a multiple of 10). This gives 90threes or 270, with 20 remaining. We now ask: 'How many threes in 21?' which has the answer 7.
- Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.

The short division method is recorded like this:

$$\begin{array}{r} 9\,7 \\ 3\overline{)2\,9\,{}^21} \end{array}$$

The carry digit '2' represents the 2 tens that have been exchanged for 20 units. In the first recording above it is written in front of the 1 to show that a total of 21 units are to be divided by 3.

The 97 written above the line represents the answer: 90 + 7,or 9 tens and 7 units.

Stage 5: Long division

The next step is to tackle HTU ÷ TU.

The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient.

Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.

How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

$$\begin{array}{r} 24\overline{)560} \\ 20 - \underline{480} \qquad 24 \times 20 \\ 80 \\ 3 \quad \underline{72} \qquad 24 \times 3 \\ 8 \\ \text{Answer: 23 R 8} \end{array}$$

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

```
        23
24) 560
   -480
    ───
     80
    -72
    ───
      8
Answer: 23 R 8
```

Date	Personnel	Comments/review
May 2011	Mr. Mawson (Maths subject Leader) and all staff	Place on Staff Policies and ensure all teachers and TAs aware of this document.
July 2012	Mr. Mawson (Maths subject Leader) and all teachers	Place on Staff Policies and ensure all teachers and TAs aware of this document.
5th February 2013	Mr. Mawson (Maths subject Leader) and all teachers	Place on Staff Policies and ensure all teachers and TAs aware of this document.
14th October 2014	Mr. Mawson (Maths subject Leader) and all teachers	Place on Staff Policies and ensure all teachers and TAs aware of this document.
30th November 2015	Mr. Mawson (Maths subject Leader)	Place on Staff Policies and ensure all teachers and TAs aware of this document.
5th December 2017	Mr. Mawson (Maths subject Leader)	Place on Staff Policies and ensure all teachers and TAs aware of this document.

Next review date: December 2019